COMMON CORE MATHEMATICS

Integration Activity

Pathway: Agribusiness

Lesson: ABR B6–10: Calculating Depreciation

Common Core State Standards for Mathematics: 9-12.F-IF.6

Domain: Interpreting Functions F-IF

Cluster: Interpret functions that arise in applications in terms of the context.

Standard: 6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specific interval. Estimate the rate of change from a graph.

Student Objective: Students will calculate depreciation using the straight-line and sum-the-digits methods, create tables and graphs representing the decreasing value of the property, and use the data to determine the average rate of change of the value over various time intervals.

BACKGROUND KNOWLEDGE for Teachers and Students

Math Concepts:

The average rate of change of a quantity over a specific interval can be found by dividing the change in the quantity by the change in time.

For example, if an item originally costs $450 and five years later has depreciated to a value of $50, the average rate of change of the value would be as follows.

$$\text{Average rate of change} = \frac{\text{Change in Value}}{\text{Change in Time}}$$

$$= \frac{-\$400}{5} = -\$80 \text{ per year}$$

This rate is negative to represent the decrease in the value of the item over time. If the quantity were increasing over time, the rate of change would be positive.

The average rate of change of a quantity can also be determined from a graph by finding the slope of a line between two given points. Slope is a measure of how steep a line is; a higher slope indicates a steeper line and a higher rate of change.

The video below explains how to find the slope of a line from a graph.

http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/slope-and-intercepts/v/graphical-slope-of-a-line

Agriculture Concepts:

Depreciation refers to the decrease in the value of an asset over time. It is important for farmers and agribusiness owners to understand depreciation so that they know the value of their assets at any given time. Depreciable assets include machinery, equipment, breeding livestock, automobiles, electronics (computers, devices, software), and agricultural structures (grain bins, farm buildings, fences). These things depreciate over time because of wear and tear from extended use and for tax deduction purposes.

Guided Practice Exercises: ANSWER KEY

1.

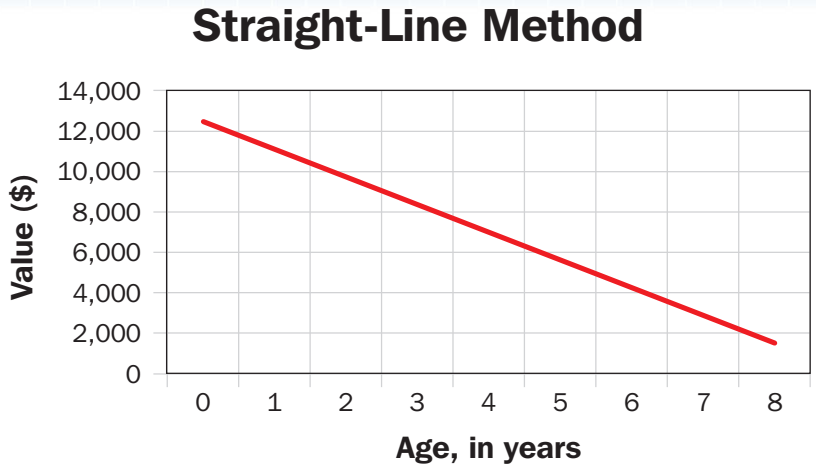
Year	Amount of Depreciation	Remaining Value
0	0	$12,500
1	$1,375	$11,125
2	$1,375	$9,750
3	$1,375	$8,375
4	$1,375	$7,000
5	$1,375	$5,625
6	$1,375	$4,250
7	$1,375	$2,875
8	$1,375	$1,500

2. Average rate of change = (1,500 – 12,500)/8 = (–11,000)/8 = –$1,375 per year

3. a. Average rate of change = (8,375 – 12,500)/3 = (–4,125)/3 = –$1,375 per year

   b. Average rate of change = (1,500 – 5,625)/3 = (–4,125)/3 = –$1,375 per year

4.



5. The value of the trailer decreases steadily over the eight-year period. For each year, the value decreases by $1,375. The graph represents this because the slope of the line is –1,375.

6.

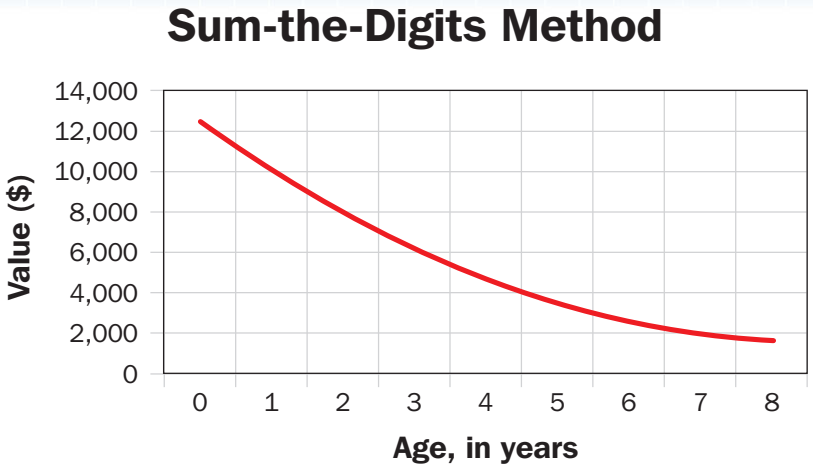
Year	Amount of Depreciation	Remaining Value
0	0	$12,500.00
1	$2,444.44	$10,055.56
2	$2,138.89	$7,916.67
3	$1,833.33	$6,083.34
4	$1,527.78	$4,555.56
5	$1,222.22	$3,333.34
6	$916.67	$2,416.67
7	$611.11	$1,805.56
8	$305.56	$1,500.00

7. Average rate of change = (1,500 – 12,500)/8 = (–11,000)/8 = –$1,375 per year

8. a. Average rate of change = (6,083.34 – 12,500)/3 = (–6,416.66)/3 = –$2,138.89 per year

   b. Average rate of change = (1,500 – 3,333,34)/3 = (–1,833.34)/3 = –$611.11 per year

9.



10. Because the graph is curved instead of straight, it is steeper in the beginning than it is at the end of the eight-year period. This tells us that the value of the trailer decreases at a higher rate in the beginning than it does toward the end of the useful life period. Our calculations also show this because the average depreciation in the first three years is $2,138.89 per year, but in the last three years the trailer depreciates an average of only $611.11 per year.

Independent Practice Exercises: ANSWER KEY

1. It is a graph of depreciation using the sum-the-digits method (because it is curved, not a straight line).
2. Student answers may vary.
   a. $23,000
   b. $4,500
3. Average rate of change = (4,500 – 23,000)/10 = (–18,500)/10 = –$1,850 per year
4. a. Average rate of change = (850 – 5,000)/5 = (–4,150)/5 = –$830 per year
   b. Average rate of change = (3,340 – 5,000)/2 = (–1,660)/2 = –$830 per year
   c. Average rate of change = (850 – 2,510)/2 = (–1,660)/2 = –$830 per year
5. (C) The bull's value decreases at a steady rate throughout the entire five-year period.

6. a. Average rate of change = (850 – 5,000)/5 = (–4,150)/5 = –$830 per year

   b. Average rate of change = (2,510 – 5,000)/2 = (–2,490)/2 = –$1,245 per year

   c. Average rate of change = (850 – 1,680)/2 = (–830)/2 = –$415 per year

7. (A) The bull's value decreases quickly initially but then slows down toward the end of the five-year period.

8. To make the investment worthwhile, the farmer must consider how much the bull will produce as well as the quality of what it is produced, since the progeny are what make the bull valuable. In particular, the farmer would want to consider the number of calves sired by this bull and the value of those calves (either for market or breeding). The farmer should also consider the cost of feeding and maintaining the bull for five years, although the income produced by the production of calves should outweigh this cost.

Name: _____

Guided Practice Exercises:

Mark purchases a livestock trailer for $12,500. The useful life is estimated to be eight years, and the salvage value is $1,500.

1. Use straight-line depreciation to complete the table below.

Year	Amount of Depreciation	Remaining Value
0	0	$12,500
1		
2		
3		
4		
5		
6		
7		
8		

2. Find the average rate of change of the value of the trailer during the first eight years.

3. Find the average rate of change of the value of the trailer during each time interval.

   a. First three years

   b. Last three years

4. Create a graph of the value of the trailer over time. Put the remaining value on the horizontal axis and the years on the vertical axis.

5. How does the graph represent the rates of change you found in questions 2 and 3?

6. Recalculate the depreciation of the trailer using the sum-the-digits method. Complete the table below.

Year	Amount of Depreciation	Remaining Value
0	0	$12,500.00
1		
2		
3		
4		
5		
6		
7		
8		

7. Based on these values, find the average rate of change of the value of the trailer during the first eight years.

8. Find the average rate of change of the value of the trailer during each time interval.

   a. First three years

   b. Last three years

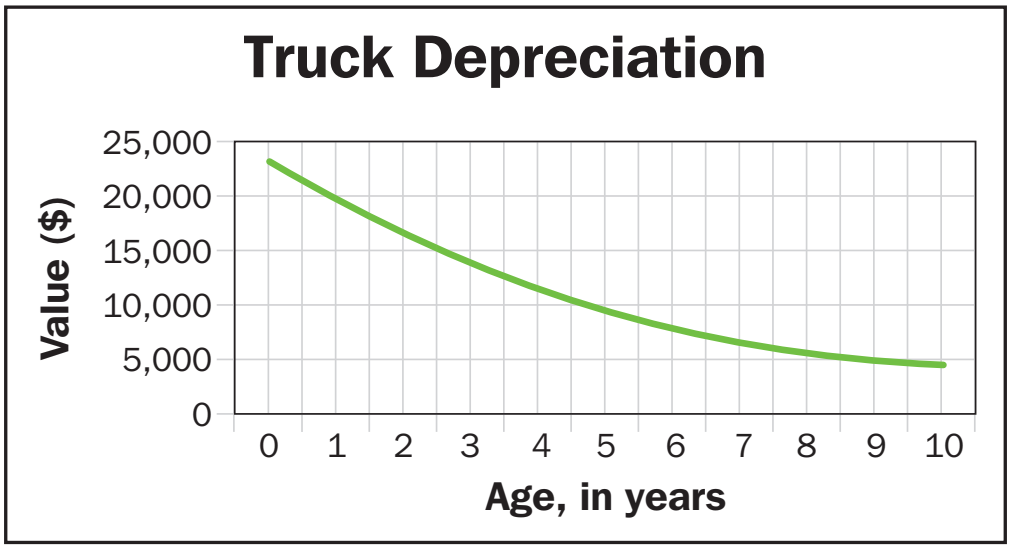
9. Create a graph of the value of the trailer over time. Put the remaining value on the horizontal axis and the years on the vertical axis.

10. How does this graph represent the rates of change you found in questions 7 and 8?

Name: _____

Independent Practice Exercises:

Below is a graph of the value of a truck over a 10-year period. Use the graph to answer questions 1 through 3.



1. Is this a graph of straight-line depreciation or depreciation using the sum-the-digits method?

2. Use the graph to estimate each of the following.

   a. Original cost of the truck

   b. Salvage value of the truck

3. Use your estimates to find the average rate of change of the value of the truck over the 10-year period.

Emily is a cattle farmer and wants to purchase a herd bull to improve the production of market-acceptable calves in her herd. The bull will cost $5,000 and has a useful life of five years. The salvage value of the bull is estimated to be $850.

Before making this purchase, Emily wants to analyze the value of the bull over the five-year period. The tables below give the value of the bull each year using straight-line and sum-the-digits depreciation.

Straight-Line Depreciation	
Year	**Value**
0	$5,000
1	$4,170
2	$3,340
3	$2,510
4	$1,680
5	$850

Sum-the-Digits Depreciation	
Year	**Value**
0	$5,000.00
1	$3,616.67
2	$2,510.00
3	$1,680.00
4	$1,126.67
5	$850.00

4. Use the straight-line depreciation values to calculate the average rate of change of the value of the bull over each time period below.

   a. Entire five-year period

   b. First two years

   c. Last two years

5. (Multiple choice) Which of the following best describes the rate of change of the value of the bull when using straight-line depreciation?

   (A) The bull's value decreases quickly initially but then slows down toward the end of the five-year period.

   (B) The bull's value decreases slightly initially but has more drastic decreases toward the end of the five-year period.

   (C) The bull's value decreases at a steady rate throughout the entire five-year period.

6. Use the sum-the-digits depreciation values to calculate the average rate of change of the value of the bull over each time period below.

   a. Entire five-year period

   b. First two years

   c. Last two years

7. (Multiple choice) Which of the following best describes the rate of change of the value of the bull when using sum-the-digits depreciation?

   (A) The bull's value decreases quickly initially but then slows down toward the end of the five-year period.

   (B) The bull's value decreases slightly initially but has more drastic decreases toward the end of the five-year period.

   (C) The bull's value decreases at a steady rate throughout the entire five-year period.

8. Although the bull will decrease in value over time, it is a critical investment for the cattle farmer. What other factors should a farmer consider when investing in a herd bull? (Remember, ultimately the farmer wants to make money!)